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STABLE SHAPE AND BROWN'S REPRESENTATION THEOREM

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This paper is based on a part of my joint paper [10] with Jack Segal. Brown's representation theorem is well-known in algebraic topology, where CW-complexes are the main objects which people look at. Just one example that I know as an application of Brown's theorem to general toplogical spaces is due to Demers [2]. He used the theorem to study topological spaces that have the shape of CW-complexes. In this paper we inroduce one interesting way of applying Brown's theorem in studying stable shape theory

Stable shape theory was first investigated by Lima [5], and various properties for compacta were obtained by Dold and Puppe [3], Henn [4], Nowak [12, 13] and Mrozik [11]. Miyata and Segal [9] then defined stable shape theory for arbitrary topological spaces, using CW-spectra, and proved the Whitehead theorem, and more recently they proved the Hurewicz theorem in this category in [10].

Throughout the paper we assume that all spaces have base points, maps are pointed maps and homotopy maps preserve base points. A space means a topological space with a base point.

1. CW-SPECTRA

Let $\mathbf{CW}_{spec}$ denote the category of CW-spectra and maps of CW-spectra. For each space X, the *suspension spectrum* $E(X)$ of X is the spectrum defined by

$$(E(X))_n = \begin{cases} S^n X & n \geq 0 \\ * & n < 0 \end{cases}$$

Here $S : \mathbf{Top} \to \mathbf{Top}$ is the functor defined by $SX = S^1 \wedge X$ for each space X and $Sf = 1_{S^1} \wedge f$ for each map $f : X \to Y$ between

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spaces where **Top** denotes the category of spaces and maps, and let $S^k = S \circ S^{k-1}$ for $k \geq 2$ and $S^1 = S$. For each map $f : X \to Y$ between CW-complexes, $E(f) : E(X) \to E(Y)$ is the map of CW-spectra defined by $(E(f))_n = S^n f : S^n X \to S^n Y$. Let $\mathbf{HCW}_{spec}$ denote the homotopy category of $\mathbf{CW}_{spec}$, i.e., the objects of $\mathbf{HCW}_{spec}$ are all CW-spectra and the morphisms are the homotopy classes of maps between CW-spectra.

For any abelian group G, let $H(G)$ denote an Eilenberg-MacLane spectrum i.e.,

$$H(G)_m = \begin{cases} H(G,m) & \text{for } m \geq 1 \\ * & \text{for } m \leq 0 \end{cases}$$

where $H(G,m)$ is an Eilenberg-MacLane complex of type (G,m). Let $\iota : S^0 \to H(\mathbb{Z})$ be a map representing $1 \in \mathbb{Z} \cong [S^0, H(\mathbb{Z})] \cong \pi_0(H(\mathbb{Z}))$. Then ι induces a natural transformation of homology theories $T_*(\iota) : \pi_*^S \to H(\mathbb{Z})_* = \tilde{H}(\ ;\mathbb{Z})$, where $\tilde{H}(\ ;\mathbb{Z})$ denotes the reduced singular homology theory with coefficients in $\mathbb{Z}$. We write h_*^S for $T_*(\iota)$ and call it the *stable Hurewicz homomorphism*. A space X is said to be *stably n-connected* if $\pi_q^S(X) = 0$ for $q \leq n$.

Theorem 1 (Stable Hurewicz theorem). *If a CW-complex X is $(n-1)$-stably connected, then the stable Hurewicz homomorphism $h_q^S : \pi_q^S(X) \to \tilde{H}_q(X;\mathbb{Z})$ is an isomorphism for $q \leq n$ and an epimorphism for $q = n+1$.*

Theorem 2 (Whitehead theorem). *Let $n \in Z \cup \{\infty\}$, let $f : E \to F$ be a map of CW-spectra, which is an n-equivalence, and suppose* $\dim E \leq n-1$ *and* $\dim F \leq n$. *Then f is a homotopy equivalence of CW-spectra.*

The reader is referred to Switzer [15] and Margolis [8] for details about CW-spectra.

2. Stable shape

In this section we recall the construction of generalized stable shape. The reader is referred to Miyata and Segal [10] for more details.

Let **HCW** denote the homotopy category of spaces having the homotopy type of CW-complexes and maps. Let $\boldsymbol{p} = (p_\lambda : \lambda \in \Lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ be an **HCW**-expansion of a space X in the sense of Mardešić and Segal [10], and let $E(\boldsymbol{X}) = (E(X_\lambda), E(p_{\lambda\lambda'}), \Lambda)$ be the inverse system in $\mathbf{HCW}_{spec}$ induced by the inverse system $\boldsymbol{X}$ in **HCW**. A morphism $\boldsymbol{e} : E(\boldsymbol{X}) \to \boldsymbol{E} = (E_a, e_{aa'}, A)$ in pro-$\mathbf{HCW}_{spec}$ is said to be a *generalized expansion* of X in $\mathbf{HCW}_{spec}$ provided the following universal property is satisfied:

(U): If $f : E(\boldsymbol{X}) \to \boldsymbol{F}$ is a morphism in **pro-HCW**$_{spec}$ then there exists a unique morphism $\boldsymbol{g} : \boldsymbol{E} \to \boldsymbol{F}$ in **pro-HCW**$_{spec}$ such that $\boldsymbol{f} = \boldsymbol{ge}$.

One should note here that the definition of a generalized expansion does not depend on the choice of the **HCW**-expansion $\boldsymbol{p}$. Also note that for any two generalized expansions $\boldsymbol{e} : E(\boldsymbol{X}) \to \boldsymbol{E}$ and $\boldsymbol{e}' : E(\boldsymbol{X}) \to \boldsymbol{E}'$ in **HCW**$_{spec}$ there exists a unique isomorphism $\boldsymbol{i} : \boldsymbol{E} \to \boldsymbol{E}'$ in **pro-HCW**$_{spec}$ (which we call the *natural isomorphism*) such that $\boldsymbol{ie} = \boldsymbol{e}'$. It is easy to see that the identity induced morphism $E(\boldsymbol{X}) \to E(\boldsymbol{X})$ is a generalized expansion of X in **HCW**$_{spec}$.

Theorem 3. *A morphism in* **pro-HCW**$_{spec}$, $\boldsymbol{e} : E(\boldsymbol{X}) \to \boldsymbol{E} = (E_a, e_{aa'}, A)$, *where* $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ *is an* **HCW**-*expansion of any space* X, *is a generalized expansion in* **HCW**$_{spec}$ *if and only if* $\boldsymbol{e}$ *is an isomorphism in* **pro-HCW**$_{spec}$.

Theorem 4. *Let* $\boldsymbol{e} : E(\boldsymbol{X}) \to \boldsymbol{E} = (E_a, e_{aa'}, A)$ *be a morphism in pro-*$\mathbf{HCW}_{spec}$ *which is represented by a morphism* (e_a, φ) *of inverse systems where* $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ *is an* **HCW**-*expansion of any space* X. *Then* $\boldsymbol{e}$ *is a generalized expansion in* **HCW**$_{spec}$ *if and only if the following two conditions are satisfied:*

(GE1): *Every morphism* $h : E(X_\lambda) \to F$ *in* **HCW**$_{spec}$ *admits* $a \in A$ *and a morphism* $g_a : E_a \to F$ *in* HCW_{spec} *such that* $hE(p_{\lambda\lambda'}) = g_a e_a E(p_{\varphi(a)\lambda'})$ *for some* $\lambda' \geq \lambda, \varphi(a)$.

(GE2): *If* $g_a, h_a : E_a \to F$ *are morphisms in* **HCW**$_{spec}$ *such that* $g_a e_a E(p_{\varphi(a)\lambda}) = h_a e_a E(p_{\varphi(a)\lambda})$ *for some* $\lambda \geq \varphi(a)$, *then there exists* $a' \geq a$ *such that* $g_a e_{aa'} = h_a e_{aa'}$.

We use generalized expansions to define the *generalized stable shape category* $\mathbf{Sh}_{spec}$ for spaces as follows: Let ob $\mathbf{Sh}_{spec}$ be the set of all spaces and CW-spectra. For any $X, Y \in$ ob $\mathbf{Sh}_{spec}$, let $\mathcal{E}_{(X,Y)}$ denote the set of all morphisms $\boldsymbol{g} : \boldsymbol{E} \to \boldsymbol{F}$ in **pro-HCW**$_{spec}$ where $\boldsymbol{E}$ is either a rudimentary system (X) (if X is a CW-spectrum) or the inverse system of CW-spectra such that $\boldsymbol{e} : E(\boldsymbol{X}) \to \boldsymbol{E} = (E_a, e_{aa'}, A)$ is a generalized expansion of X in **HCW**$_{spec}$ (if X is a space), and similarly for $\boldsymbol{F}$. We define an equivalence relation $\sim$ on $\mathcal{E}_{(X,Y)}$ as follows: for $\boldsymbol{g} : \boldsymbol{E} \to \boldsymbol{F}$ and $\boldsymbol{g}' : \boldsymbol{E}' \to \boldsymbol{F}'$ in $\mathcal{E}_{(X,Y)}$, $\boldsymbol{g} \sim \boldsymbol{g}'$ if and only if $\boldsymbol{jg} = \boldsymbol{g}'\boldsymbol{i}$ in **pro-HCW**$_{spec}$ where $\boldsymbol{i} : \boldsymbol{E} \to \boldsymbol{E}'$ and $\boldsymbol{j} : \boldsymbol{F} \to \boldsymbol{F}'$ are the natural isomorphisms. We define a morphism from X to Y as each equivalence class of $\mathcal{E}_{(X,Y)}$, and hence the set of morphisms from X to Y, $\mathbf{Sh}_{spec}(X,Y) = \mathcal{E}_{(X,Y)}/\sim$. We write $Sh_{spec}(X) = Sh_{spec}(Y)$ provided X is equivalent to Y in $\mathbf{Sh}_{spec}$. The stable shape category for compacta defined by Dold and Puppe [3] and Henn [4] can be embedded in $\mathbf{Sh}_{spec}$. Let $\mathbf{Sh}$ denote the pointed shape category for spaces in the sense of Mardešić and Segal [10]. We write $Sh(X) = Sh(Y)$ provided X is equivalent to Y in $\mathbf{Sh}$. Then there exists a functor $\Xi : \mathbf{Sh} \to \mathbf{Sh}_{spec}$ and we have

Theorem 5. *For any spaces X and Y, if $Sh(S^kX) = Sh(S^kY)$ for some $k \geq 0$ then $Sh_{spec}(X) = Sh_{spec}(Y)$. Conversely, for any compact Hausdorff spaces X and Y with finite shape dimension (see Mardešić and Segal* [10, II, §1]*), if $Sh_{spec}(X) = Sh_{spec}(Y)$, then $Sh(S^kX) = Sh(S^kY)$ for some $k \geq 0$.*

Example. There exists a finite polyhedron P with $\pi_1(P) \neq 0$ but whose suspension SP is contractible. Indeed, let P be the homological 3-sphere with an open 3-simplex removed from its triangulation. Then $Sh(P) \neq Sh(*)$ but $Sh_{spec}(P) = Sh_{spec}(*)$. There is also a non-polyhedral example. Let X be the 1-dimensional acyclic continuum ("figure eight"-like continuum) described by Case and Chamberlin [1]. Then X is non-movable, so that $Sh(X) \neq Sh(*)$, but its suspension SX is of trivial shape i.e., $Sh(SX) = Sh(*)$ (see Mardešić [6]), so that $Sh_{spec}(X) = Sh_{spec}(*)$.

3. Whitehead and Hurewicz theorems

In order to state Whitehead theorems in $\mathbf{Sh}_{spec}$, we need notions of dimension in this category. For $k, n \in Z$ with $k \leq n$ and for every space X, we say the *stable shape dimension* $k \leq \mathrm{sd}_{spec}X \leq n$ if whenever $\boldsymbol{e} : E(\boldsymbol{X}) \to \boldsymbol{E} = (E_a, e_{aa'}, A)$ is a generalized expansion in $\mathbf{HCW}_{spec}$, then every $a \in A$ admits $a' \geq a$ such that $e_{aa'}$ factors in $\mathbf{HCW}_{spec}$ through a CW-spectrum F such that i) $\dim F \leq n$ and ii) whenever $e \neq *$ is a cell of F, $\dim e \geq k$. For $k, n \in Z$, we say the *stable shape dimension* $k \leq \mathrm{sd}_{spec}X \leq \infty$ (respectively, $-\infty \leq \mathrm{sd}_{spec}X \leq n$) if whenever $\boldsymbol{e} : E(\boldsymbol{X}) \to \boldsymbol{E} = (E_a, e_{aa'}, A)$ is a generalized expansion in HCW_{spec}, then every $a \in A$ admits $a' \geq a$ such that $e_{aa'}$ factors in $\mathbf{HCW}_{spec}$ through a CW-spectrum F such that whenever $e \neq *$ is a cell of F, $\dim e \geq k$ (respectively, $\dim F \leq n$).

For $-\infty < k \leq n < \infty$, it is obvious that $k \leq \mathrm{sd}_{spec}X \leq n$ implies $k \leq \mathrm{sd}_{spec}X \leq n+1$ and $k-1 \leq \mathrm{sd}_{spec}X \leq n$, and that $k \leq \mathrm{sd}_{spec}X \leq n$ implies $k \leq \mathrm{sd}_{spec}X \leq \infty$ and $-\infty \leq \mathrm{sd}_{spec}X \leq n$.

Those notions are invariant in $\mathbf{Sh}_{spec}$, and characterizations of stable shape dimension are discussed in [10].

Theorem 6. *For every space X of $sdX < \infty$, $0 \leq sd_{spec}X \leq sdX$.*

Example. Let X be the 1-dimensional acyclic continuum of Case and Chamberlin [1]. Then $\mathrm{sd}X = 1$, but $0 \leq \mathrm{sd}_{spec}X \leq 0$ as $Sh_{spec}(X) = Sh_{spec}(*)$.

There also exists a compactum X such that

$$\mathrm{sd}X = \infty \text{ and } -\infty \leq \mathrm{sd}_{spec}X \leq n \text{ for some } n \in Z$$

The reader should see [14, p. 46] where a movable continuum X with infinite sd such that the suspension of X has trivial shape is given. More specifically, $X = \prod_{i=1}^{\infty} P_i$ where P_i is the complement of an open ball in the Poincaré manifold.

Now we wish to Čech-extend the definition of π_n on $\mathbf{HCW}_{spec}$ over $\mathbf{Sh}_{spec}$. For each space X, the *n-th stable pro-homotopy group* $pro\text{-}\pi_n^S(X)$ is defined as the inverse system $\pi_n(E(\boldsymbol{X})) = (\pi_n(E_a), \pi_n(e_{aa'}), A)$, where $e : E(\boldsymbol{X}) \to \boldsymbol{E} = (E_a, e_{aa'}, A)$ is a generalized $\mathbf{HCW}^s_{spec}$-expansion of $E(X)$. This is well-defined up to an isomorphism in pro-groups. Then the *n-th stable shape group* $\check{\pi}_n^S(X)$ is defined as the limit group $\lim pro\text{-}\pi_n(\boldsymbol{E})$.

For each morphism $G : X \to Y$ in $\mathbf{Sh}_{spec}$, we define the morphism in pro-groups $pro\text{-}\pi_n^S(G) : pro\text{-}\pi_n^S(X) \to pro\text{-}\pi_n^S(Y)$ as $pro\text{-}\pi_n(\boldsymbol{g}) : \pi_n(\boldsymbol{E}) \to \pi_n(\boldsymbol{F})$, where $\boldsymbol{e} : E(X) \to \boldsymbol{E}$ and $\boldsymbol{f} : E(Y) \to \boldsymbol{F}$ are $\mathbf{HCW}_{spec}$-expansions of X and Y, respectively, and $\boldsymbol{g} : \boldsymbol{E} \to \boldsymbol{F}$ is a representative of G. This is well-defined up to an isomorphism in pro-groups. It is a routine to check $pro\text{-}\pi_n^S$ is a functor from $\mathbf{Sh}_{spec}$ to $\mathbf{pro\text{-}Gp}$ and that $\check{\pi}_n^S$ is a functor from $\mathbf{Sh}_{spec}$ to $\mathbf{Gp}$.

A morphism $G : X \to Y$ in $\mathbf{Sh}_{spec}$ is said to be an *n-equivalence* if the induced morphism in pro-groups $pro\text{-}\pi_k^S(G) : pro\text{-}\pi_k^S(X) \to pro\text{-}\pi_k^S(Y)$ is an isomorphism for $k = 0, \ldots, n-1$ and an epimorphism for $k = n$.

Now we are ready to state the Whitehead theorems in Sh_{spec}.

Theorem 7. *Let $G : X \to Y$ be a morphism in $\mathbf{Sh}_{spec}$, which is an n-equivalence. Suppose that $-\infty \leq sd_{spec}X \leq n-1$ and $k \leq sd_{spec}Y \leq n$ $(k, n \in Z)$. Then G is an isomorphism in $\mathbf{Sh}_{spec}$.*

Remark. The infinite-dimensionality of the above theorems cannot be omitted. Recall the example in Mardešić and Segal [7, Example 1, p.153].

For $n \in \mathbb{Z}$, a space X is said to be *stable shape n-connected* if $pro\text{-}\pi_q^S(X) = 0$ for $q \leq n$.

Theorem 8. *If a space X is stable shape $(n-1)$-connected for $n \geq 1$, then the stable Hurewicz homomorphism $pro\text{-}h_q^S : pro\text{-}\pi_q^S(X) \to pro\text{-}\bar{H}_q(X; \mathbb{Z})$ is an isomorphism for $q \leq n$ and an epimorphism for $q = n+1$.*

4. Brown's representation theorem

Let $\mathbf{HCW}^f_{spec}$ denote the full subcategory of $\mathbf{HCW}_{spec}$ whose objects are all finite CW-spectra. For each CW-spectrum E, let E_* and E^* denote the homology and cohomology theories associated with E, respectively. We now recall the following version of Brown's representation theorem (see Switzer [15, Theorems 14.35 and 14.36] and Margolis [8, Section 4.3]).

Theorem 9. ***i***) *Let h_* be a homology theory on $\mathbf{HCW}^f_{spec}$. Then there exist a CW-spectrum E and a natural equivalence $\tau_f : E_* \to h_*$.*

ii) *Let h_* be a homology theory on $\mathbf{HCW}_{spec}$ with the following property:*

(D): *For any CW-spectrum G, the inclusion maps $i_\alpha : G_\alpha \hookrightarrow G$ of finite subspectra G_α into G induce the isomorphism:*

$$\tau = \operatorname*{colim}_{\alpha} i_{\alpha *} : \operatorname*{colim}_{\alpha} h_q(G_\alpha) \longrightarrow h_q(G) \text{ for each } q \in \mathbb{Z}$$

Then there exist a CW-spectrum E and a natural equivalence $\tau : E_* \to h_*$ which extends the natural equivalence τ_f on $\mathbf{HCW}^f_{spec}$ of (i).

iii) *Let h_* and h'_* be homology theories on $\mathbf{HCW}^f_{spec}$, and let E and E' be the CW-spectra corresponding to h_* and h'_*, respectively. Then each natural transformation $T : h_0 \to h'_0$ admits a map $f : E \to E'$ such that the following diagram commutes for each finite CW-spectrum G:*

$$\begin{array}{ccc} h_0(G) & \xrightarrow{T(G)} & h'_0(G) \\ {\scriptstyle \tau(G)}\uparrow & & \uparrow{\scriptstyle \tau'(G)} \\ [S^0, E \wedge G] & \xrightarrow{T_f(G)} & [S^0, E' \wedge G] \end{array}$$

where T_f is the natural transformation induced by f. Moreover, such an f is unique up to weak homotopy.

iv) *The CW-spectra E in (i) and (ii) are unique up to homotopy.*

5. An application of Brown's representation theorem in stable shape

Lemma 10. *For any X, $Y \in \operatorname{ob}\mathbf{Sh}_{spec}$, $\mathbf{Sh}_{spec}(X, Y)$ has the structure of an abelian group.*

Let Σ also denote the suspension functor on $\mathbf{Sh}_{spec}$, and as before, let $\Sigma^{k+1} = \Sigma \circ \Sigma^k$ and $\Sigma^1 = \Sigma$.

Lemma 11. *Let* $X, Y \in \mathrm{ob}\,\mathbf{Sh}_{spec}$. *Then there is a natural bijection:*

$$\Sigma : \mathbf{Sh}_{spec}(X, Y) \to \mathbf{Sh}_{spec}(\Sigma X, \Sigma Y)$$

Let **Ab** denote the category of abelian groups and homomorphisms. For each $q \in \mathbb{Z}$ and for each space Z, we define the covariant functor $Z_q : \mathbf{HCW}_{spec} \to \mathbf{Ab}$ as follows:

$$Z_q = \begin{cases} \mathbf{Sh}_{spec}(\Sigma^q Z, \_) & \text{for } q \geq 0 \\ \mathbf{Sh}_{spec}(Z, \Sigma^{-q}\,\_) & \text{for } q < 0 \end{cases}$$

and also define the natural equivalence $\sigma_q : Z_q \to Z_{q+1} \circ \Sigma$ as follows: for each CW-spectrum G,

$$\sigma_q(G) : \begin{cases} Z_q(G) \xrightarrow{\Sigma} Z_{q+1}(\Sigma G) & \text{for } q \geq 0 \\ Z_q(G) \xrightarrow{=} Z_{q+1}(\Sigma G) & \text{for } q < 0 \end{cases}$$

Lemma 12. *For each* $Z \in \mathrm{ob}\,\mathbf{Sh}_{spec}$, $Z_* = (Z_q, \sigma_q : q \in \mathbb{Z})$ *forms a homology theory on* $\mathbf{HCW}_{spec}$.

Lemma 13. *For each compact Hausdorff space* Z, *the homology theory* Z_* *has the property D.*

Lemma 14. *For any* $Z,\ Z' \in \mathrm{ob}\,\mathbf{Sh}_{spec}$, Z_* *is naturally equivalent to* Z'_* *on* $\mathbf{HCW}_{spec}$ *if and only if* $Sh_{spec}(Z) = Sh_{spec}(Z')$.

Theorem 15. *Let* $\mathbf{Comp}_{spec}$ *denote the full subcategory of* $\mathbf{Sh}_{spec}$ *whose objects are all compact Hausdorff spaces, and let* $\mathbf{WCW}_{spec}$ *denote the category of CW-spectra and weak homotopy equivalence classes.*

i) There exists a contravariant functor $\Pi : \mathbf{Sh}_{spec} \to \mathbf{WCW}_{spec}$.

ii) The restriction $\Pi|\mathbf{Comp}_{spec} : \mathbf{Comp}_{spec} \to \mathbf{WCW}_{spec}$ *is a full embedding.*

Proof: (outline) For each $Z \in \mathrm{ob}\,\mathbf{Sh}_{spec}$, Z_* forms a homology theory on $\mathbf{HCW}^f_{spec}$. Thus there exist a unique (up to homotopy) $E \in \mathrm{ob}\,\mathbf{HCW}_{spec}$ and a natural equivalence $\tau_f : E_* \to Z_*$ on $\mathbf{HCW}^f_{spec}$. Let $\Pi(Z)$ be the CW-spectrum E. For each $\varphi \in \mathbf{Sh}_{spec}(Z, Z')$, there exists an induced natural transformation $\varphi^* : \mathbf{Sh}_{spec}(Z', \_) \to \mathbf{Sh}_{spec}(Z, \_)$ on $\mathbf{HCW}^f_{spec}$. Then Brown's theorem implies that there exists a unique (up to weak homotopy) map $f : E' \to E$ such that the following diagram commutes on $\mathbf{HCW}^f_{spec}$:

$$\begin{array}{ccc} \mathbf{Sh}_{spec}(Z', \ \_\ ) & \xrightarrow{\varphi^*} & \mathbf{Sh}_{spec}(Z, \ \_\ ) \\ {\scriptstyle \tau}\uparrow & & \uparrow{\scriptstyle \tau} \\ [S^0, E' \wedge \ \_\ ] & \xrightarrow{T_f} & [S^0, E \wedge \ \_\ ] \end{array}$$

Let $\Pi(\varphi)$ be the map f. Then $\Pi : \mathbf{Sh}_{spec} \to \mathbf{WCW}_{spec}$ forms a contravariant functor.

Suppose now that $Z, Z' \in \mathrm{ob}\,\mathbf{Comp}_{spec}$ are such that $\Pi(Z) = \Pi(Z')$ in $\mathbf{WCW}_{spec}$. Then there is a natural equivalence $Z_* \to Z'_*$ on $\mathbf{HCW}_{spec}$, so $Sh_{spec}(Z) = Sh_{spec}(Z')$. Let $f : E' \to E$ be a map where $E = \Pi(Z)$ and $E' = \Pi(Z')$. Then, since Z_* and Z'_* are homology theories on $\mathbf{HCW}_{spec}$ with property (D), this induces a natural transformation $T : \mathbf{Sh}_{spec}(Z', \ \_\ ) \to \mathbf{Sh}_{spec}(Z, \ \_\ )$ on $\mathbf{HCW}_{spec}$ such that the following diagram commutes on $\mathbf{HCW}_{spec}$:

$$\begin{array}{ccc} \mathbf{Sh}_{spec}(Z', \ \_\ ) & \xrightarrow{T} & \mathbf{Sh}_{spec}(Z, \ \_\ ) \\ {\scriptstyle \tau'}\uparrow & & \uparrow{\scriptstyle \tau} \\ [S^0, E' \wedge \ \_\ ] & \xrightarrow{T_f} & [S^0, E \wedge \ \_\ ] \end{array}$$

So, there is a unique $\varphi \in \mathbf{Sh}_{spec}(Z, Z')$ such that $\varphi^* = T : \mathbf{Sh}_{spec}(Z', \ \_\ ) \to \mathbf{Sh}_{spec}(Z, \ \_\ )$ on $\mathbf{HCW}_{spec}$. If $f, f' : E' \to E$ are weakly homotopic to each other, then $T_f = T_{f'}$. This shows that there is a contravariant functor Π' from the range of Π onto $\mathbf{Comp}_{spec}$ which defines the inverse of Π. □

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